GEOMETRY

SUMMER PACKET 2020 – 2021

CONWELL-EGAN CATHOLIC HIGH SCHOOL

All students who are enrolled in a mathematics course for the 2020 – 2021 school year have a Mathematics Summer Packet to complete.

Within the first few days of your Geometry course, you will be assessed on the prerequisite skills outline in this packet. This summer assignment is a review and exploration of key skills that are necessary for success in your 2020 – 2021 mathematics course as well as future high school mathematics courses.

The assessment will count as a full test grade in your first quarter average.

All summer packets are due on September 14, 2020

VARIABLE EXPRESSIONS

Example 4 Write Verbal Expressions

Write a verbal expression for each algebraic expression.

a. $4m^3$

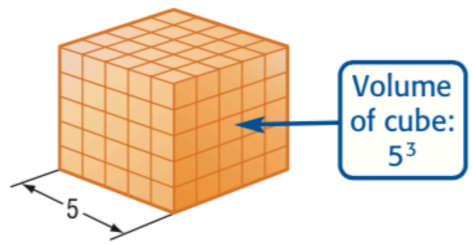
the product of 4 and m to the third power

b. $c^2 + 21d$

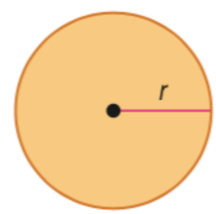
the sum of c squared and 21 times d

c. 5^3

five to the third power or five cubed



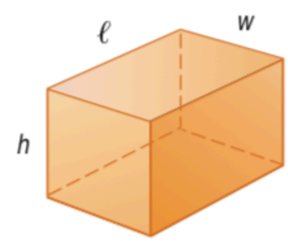
20. **GEOMETRY** The area of a circle can be found by multiplying the number π by the square of the radius. If the radius of a circle is r, write an expression that represents the area of the circle.



44. **GEOMETRY** The surface area of a rectangular prism is the sum of:

- the product of twice the length ℓ and the width w,
- the product of twice the length and the height h, and
- the product of twice the width and the height.

Write an expression that represents the surface area of a prism.



46. **CRITICAL THINKING** In the square, the variable a represents a positive whole number. Find the value of a such that the area and the perimeter of the square are the same.

EVALUATING EXPRESSIONS

Example 5 Use Algebraic Expressions

ARCHITECTURE **The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. The area of its base is 360,000 square feet, and it is 321 feet high. The volume of any pyramid is one third of the product of the area of the base B and its height h.**

a. Write an expression that represents the volume of a pyramid.

$$\underbrace{\frac{1}{3}}_{\text{one third}} \quad \underbrace{\times}_{\text{of}} \quad \underbrace{(B \cdot h)}_{\text{the product of area of base and height}} \quad \text{or } \frac{1}{3}Bh$$

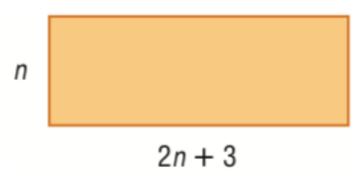
b. Find the volume of the Pyramid Arena.

Evaluate $\frac{1}{3}(Bh)$ for $B = 360{,}000$ and $h = 321$.

$$\begin{aligned}
\frac{1}{3}(Bh) &= \frac{1}{3}(360{,}000 \cdot 321) && B = 360{,}000 \text{ and } h = 321\\
&= \frac{1}{3}(115{,}560{,}000) && \text{Multiply 360,000 by 321.}\\
&= \frac{115{,}560{,}000}{3} && \text{Multiply } \tfrac{1}{3} \text{ by 115,560,000.}\\
&= 38{,}520{,}000 && \text{Divide 115,560,000 by 3.}
\end{aligned}$$

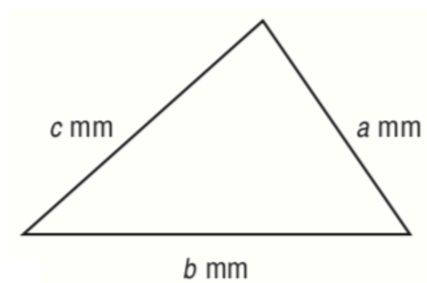
The volume of the Pyramid Arena is 38,520,000 cubic feet.

29. **GEOMETRY** Find the area of the rectangle when $n = 4$ centimeters.



46. Find the perimeter of the triangle using the formula $P = a + b + c$ if $a = 10$, $b = 12$, and $c = 17$.

Ⓐ 39 mm Ⓑ 19.5 mm
Ⓒ 60 mm Ⓓ 78 mm



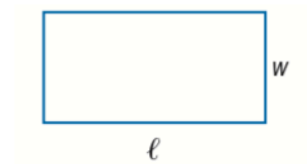
Ⓓ 39

WRITING EQUATIONS

Example 3 Write a Formula

Translate the sentence into a formula.

The perimeter of a rectangle equals two times the length plus two times the width.



Words Perimeter equals two times the length plus two times the width.

Variables Let P = perimeter, ℓ = length, and w = width.

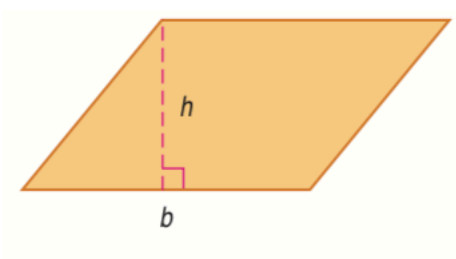
Formula

$$\overbrace{P}^{\text{Perimeter}} \quad \overbrace{=}^{\text{equals}} \quad \overbrace{2\ell}^{\text{two times the length}} \quad \overbrace{+}^{\text{plus}} \quad \overbrace{2w}^{\text{two times the width.}}$$

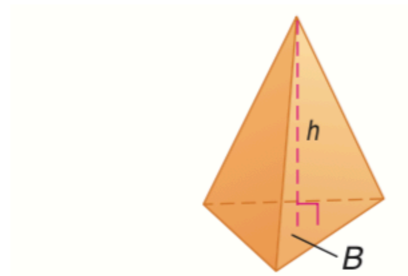
The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

Translate each sentence into a formula.

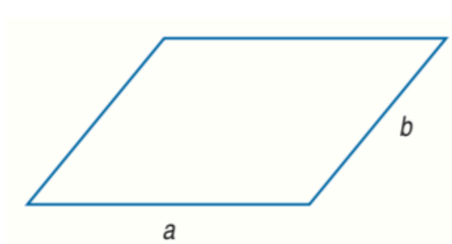
23. The area A of a parallelogram is the base b times the height h.



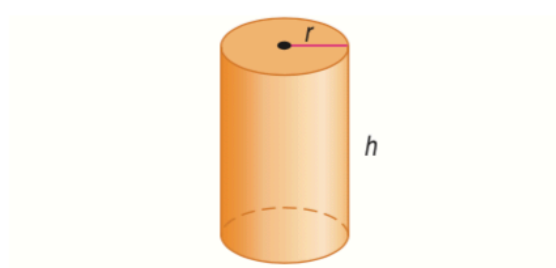
24. The volume V of a pyramid is one-third times the product of the area of the base B and its height h.



25. The perimeter P of a parallelogram is twice the sum of the lengths of the two adjacent sides, a and b.

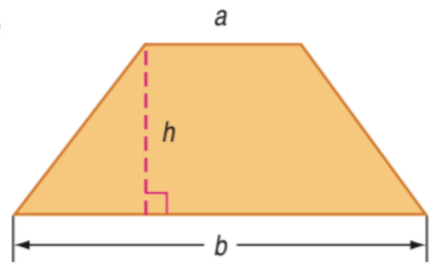


26. The volume V of a cylinder equals the product of π, the square of the radius r of the base, and the height.

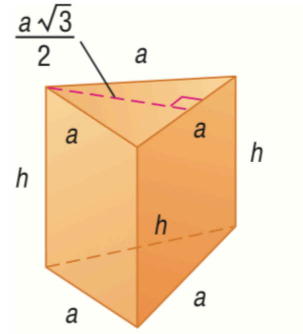


27. In a right triangle, the square of the measure of the hypotenuse c is equal to the sum of the squares of the measures of the legs, a and b.

37. **GEOMETRY** If a and b represent the lengths of the bases of a trapezoid and h represents its height, then the formula for the area A of the trapezoid is $A = \frac{1}{2}h(a + b)$. Write the formula in words.



52. **CRITICAL THINKING** The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism at the right.



GEOMETRY **For Exercises 41 and 42, use the following information.**
The volume V of a cone equals one-third times the product of π, the square of the radius r of the base, and the height h.

41. Write the formula for the volume of a cone.

42. Find the volume of a cone if r is 10 centimeters and h is 30 centimeters.

GEOMETRY **For Exercises 43 and 44, use the following information.**
The volume V of a sphere is four-thirds times π times the radius r of the sphere cubed.

43. Write a formula for the volume of a sphere.

44. Find the volume of a sphere if r is 4 inches.

SOLVING EQUATIONS

Example 2 Solve an Equation with Grouping Symbols

Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Then check your solution.

$4(2r - 8) = \frac{1}{7}(49r + 70)$	Original equation
$8r - 32 = 7r + 10$	Distributive Property
$8r - 32 - 7r = 7r + 10 - 7r$	Subtract $7r$ from each side.
$r - 32 = 10$	Simplify.
$r - 32 + 32 = 10 + 32$	Add 32 to each side.
$r = 42$	Simplify.

CHECK

$4(2r - 8) = \frac{1}{7}(49r + 70)$	Original equation
$4[2(42) - 8] \stackrel{?}{=} \frac{1}{7}[49(42) + 70]$	Substitute 42 for r.
$4(84 - 8) \stackrel{?}{=} \frac{1}{7}(2058 + 70)$	Multiply.
$4(76) \stackrel{?}{=} \frac{1}{7}(2128)$	Add and subtract.
$304 = 304$ ✓	

The solution is 42.

4. Justify each step.

$\frac{4 - 2d}{5} + 3 = 9$	
$\frac{4 - 2d}{5} + 3 - 3 = 9 - 3$	**a.** ___?___
$\frac{4 - 2d}{5} = 6$	**b.** ___?___
$\frac{4 - 2d}{5}(5) = 6(5)$	**c.** ___?___
$4 - 2d = 30$	**d.** ___?___
$4 - 2d - 4 = 30 - 4$	**e.** ___?___
$-2d = 26$	**f.** ___?___
$\frac{-2d}{-2} = \frac{26}{-2}$	**g.** ___?___
$d = -13$	**h.** ___?___

4. Justify each step.

$6n + 7 = 8n - 13$

$6n + 7 - 6n = 8n - 13 - 6n$ **a.** ?

$7 = 2n - 13$ **b.** ?

$7 + 13 = 2n - 13 + 13$ **c.** ?

$20 = 2n$ **d.** ?

$\frac{20}{2} = \frac{2n}{2}$ **e.** ?

$10 = n$ **f.** ?

Justify each step.

14. $\frac{3m - 2}{5} = \frac{7}{10}$

$\frac{3m - 2}{5}(10) = \frac{7}{10}(10)$ **a.** ?

$(3m - 2)2 = 7$ **b.** ?

$6m - 4 = 7$ **c.** ?

$6m - 4 + 4 = 7 + 4$ **d.** ?

$6m = 11$ **e.** ?

$\frac{6m}{6} = \frac{11}{6}$ **f.** ?

$m = 1\frac{5}{6}$ **g.** ?

15. $v + 9 = 7v + 9$

$v + 9 - v = 7v + 9 - v$ **a.** ?

$9 = 6v + 9$ **b.** ?

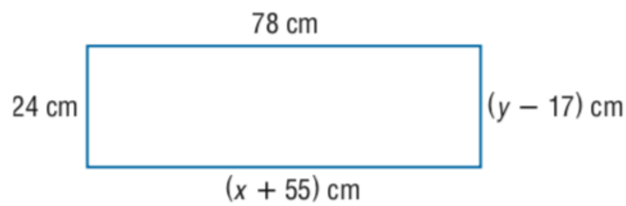
$9 - 9 = 6v + 9 - 9$ **c.** ?

$0 = 6v$ **d.** ?

$\frac{0}{6} = \frac{6v}{6}$ **e.** ?

$0 = v$ **f.** ?

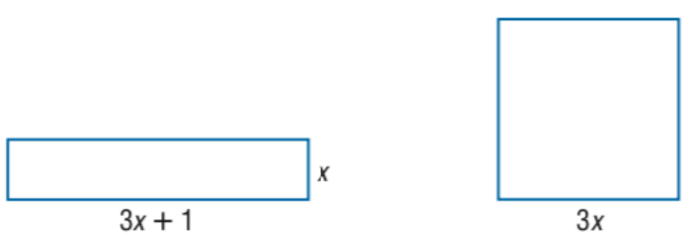
GEOMETRY **For Exercises 41 and 42, use the rectangle at the right.**



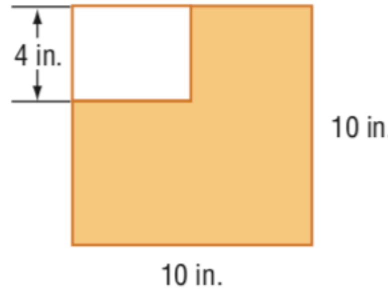
41. Write an equation you could use to solve for x and then solve for x.

42. Write an equation you could use to solve for y and then solve for y.

47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.



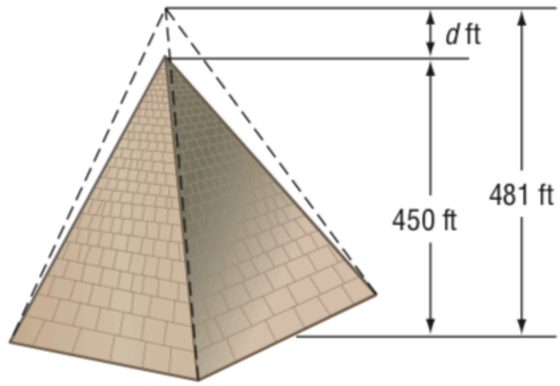
54. **GEOMETRY** A rectangle is cut from the corner of a 10-inch by 10-inch of paper. The area of the remaining piece of paper is $\frac{4}{5}$ of the area of the original piece of paper. If the width of the rectangle removed from the paper is 4 inches, what is the length of the rectangle?



HISTORY **For Exercises 56 and 57, use the following information.**
Over the years, the height of the Great Pyramid at Giza, Egypt, has decreased.

56. Write an addition equation to represent the situation.

57. What was the decrease in the height of the pyramid?



Source: *World Book Encyclopedia*

RATIO AND PROPORTION

Example 2 Use Cross Products

Use cross products to determine whether each pair of ratios form a proportion.

a. $\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4}$ Write the equation.

$0.4(1.4) \stackrel{?}{=} 0.8(0.7)$ Find the cross products.

$0.56 = 0.56$ Simplify.

The cross products are equal, so $\frac{0.4}{0.8} = \frac{0.7}{1.4}$. Since the ratios are equal, they form a proportion.

b. $\frac{6}{8}, \frac{24}{28}$

$\frac{6}{8} \stackrel{?}{=} \frac{24}{28}$ Write the equation.

$6(28) \stackrel{?}{=} 8(24)$ Find the cross products.

$168 \neq 192$ Simplify.

The cross products are not equal, so $\frac{6}{8} \neq \frac{24}{28}$. The ratios do not form a proportion.

Example 3 Solve a Proportion

Solve the proportion $\frac{n}{15} = \frac{24}{16}$.

$\frac{n}{15} = \frac{24}{16}$	Original equation
$16(n) = 15(24)$	Find the cross products.
$16n = 360$	Simplify.
$\frac{16n}{16} = \frac{360}{16}$	Divide each side by 16.
$n = 22.5$	Simplify.

Example 4 Use Rates

BICYCLING **Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?**

Explore Let m represent the number of miles Trent can ride in 6 hours.

Plan Write a proportion for the problem.

$$\begin{array}{r} \text{miles} \rightarrow \\ \text{hours} \rightarrow \end{array} \frac{30}{4} = \frac{m}{6} \begin{array}{l} \leftarrow \text{miles} \\ \leftarrow \text{hours} \end{array}$$

Solve

$\frac{30}{4} = \frac{m}{6}$	Original proportion
$30(6) = 4(m)$	Find the cross products.
$180 = 4m$	Simplify.
$\frac{180}{4} = \frac{4m}{4}$	Divide each side by 4.
$45 = m$	Simplify.

Examine If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6×7.5 or 45 miles. The answer is correct.

Since the rates are equal, they form a proportion. So, Trent can ride 45 miles in 6 hours.

Example 5 Use a Scale Drawing

CRATER LAKE **The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance between these two places?**

Let d represent the actual distance.

$$\begin{array}{rl} \text{scale} \rightarrow & \dfrac{2}{9} = \dfrac{1\frac{3}{4}}{d} \leftarrow \text{scale} \\ \text{actual} \rightarrow & \qquad\qquad \leftarrow \text{actual} \end{array}$$

$2(d) = 9\left(1\frac{3}{4}\right)$ Find the cross products.

$2d = \frac{63}{4}$ Simplify.

$2d \div 2 = \frac{63}{4} \div 2$ Divide each side by 2.

$d = \frac{63}{8}$ or $7\frac{7}{8}$ Simplify.

The actual distance is about $7\frac{7}{8}$ miles.

Solve each proportion.

1. $\frac{x}{5} = \frac{3}{4}$
2. $\frac{x}{9} = \frac{0.24}{3}$
3. $\frac{1}{y-3} = \frac{3}{y-5}$
4. $\frac{6p-2}{7} = \frac{5p+7}{8}$
5. $\frac{w-5}{4} = \frac{w+3}{3}$
6. $\frac{4n+5}{5} = \frac{2n+7}{7}$

31. **WORK** Seth earns \$152 in 4 days. At that rate, how many days will it take him to earn \$532?

32. **DRIVING** Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?

33. **BLUEPRINTS** A blueprint for a house states that 2.5 inches equals 10 feet. If the length of a wall is 12 feet, how long is the wall in the blueprint?

34. **MODELS** A collector's model racecar is scaled so that 1 inch on the model equals $6\frac{1}{4}$ feet on the actual car. If the model is $\frac{2}{3}$ inch high, how high is the actual car?

35. **PETS** A research study shows that three out of every twenty pet owners got their pet from a breeder. Of the 122 animals cared for by a veterinarian, how many would you expect to have been bought from a breeder?

TRANSFORMING FORMULAS

Example 1 Solve an Equation for a Specific Variable

Solve $3x - 4y = 7$ for y.

$3x - 4y = 7$	Original equation
$3x - 4y - 3x = 7 - 3x$	Subtract $3x$ from each side.
$-4y = 7 - 3x$	Simplify.
$\frac{-4y}{-4} = \frac{7 - 3x}{-4}$	Divide each side by -4.
$y = \frac{7 - 3x}{-4}$ or $\frac{3x - 7}{4}$	Simplify.

The value of y is $\frac{3x - 7}{4}$.

Example 2 Solve an Equation for a Specific Variable

Solve $2m - t = sm + 5$ for m.

$2m - t = sm + 5$	Original equation
$2m - t - sm = sm + 5 - sm$	Subtract sm from each side.
$2m - t - sm = 5$	Simplify.
$2m - t - sm + t = 5 + t$	Add t to each side.
$2m - sm = 5 + t$	Simplify.
$m(2 - s) = 5 + t$	Use the Distributive Property.
$\frac{m(2 - s)}{2 - s} = \frac{5 + t}{2 - s}$	Divide each side by $2 - s$.
$m = \frac{5 + t}{2 - s}$	Simplify.

The value of m is $\frac{5 + t}{2 - s}$. Since division by 0 is undefined, $2 - s \neq 0$ or $s \neq 2$.

Example 3 Use a Formula to Solve Problems

WEATHER **Use the information about the Kansas City hailstorm at the left. The formula for the circumference of a circle is $C = 2\pi r$, where C represents circumference and r represent radius.**

a. Solve the formula for r.

$C = 2\pi r$ Formula for circumference

$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ Divide each side by 2π.

$\frac{C}{2\pi} = r$ Simplify.

b. Find the radius of one of the large hailstones that fell on Kansas City in 1898.

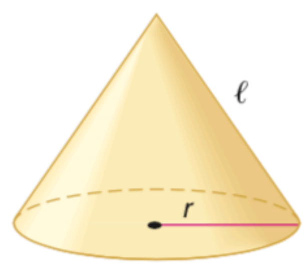
$\frac{C}{2\pi} = r$ Formula for radius

$\frac{9.5}{2\pi} = r$ $C = 9.5$

$1.5 \approx r$ The largest hailstones had a radius of about 1.5 inches.

EXAMPLE Solve for a Variable

6 **GEOMETRY** **The formula for the surface area S of a cone is $S = \pi r\ell + \pi r^2$, where ℓ is the slant height of the cone and r is the radius of the base. Solve the formula for ℓ.**



$S = \pi r\ell + \pi r^2$ Surface area formula

$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$ Subtract πr^2 from each side.

$S - \pi r^2 = \pi r\ell$ Simplify.

$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r}$ Divide each side by πr.

$\frac{S - \pi r^2}{\pi r} = \ell$ Simplify.

Example 4 Use Dimensional Analysis

PHYSICAL SCIENCE **The formula $s = \frac{1}{2}at^2$ represents the distance s that a free-falling object will fall near a planet or the moon in a given time t. In the formula, a represents the acceleration due to gravity.**

a. Solve the formula for a.

$s = \frac{1}{2}at^2$ — Original formula

$\frac{2}{t^2}(s) = \frac{2}{t^2}\left(\frac{1}{2}at^2\right)$ — Multiply each side by $\frac{2}{t^2}$.

$\frac{2s}{t^2} = a$ — Simplify.

b. A free-falling object near the moon drops 20.5 meters in 5 seconds. What is the value of a for the moon?

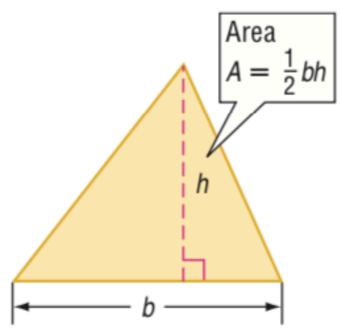
$a = \frac{2s}{t^2}$ — Formula for a

$a = \frac{2(20.5m)}{(5s)^2}$ — $s = 20.5m$ and $t = 5s$.

$a = \frac{1.64m}{s^2}$ or $1.64\ m/s^2$ — Use a calculator.

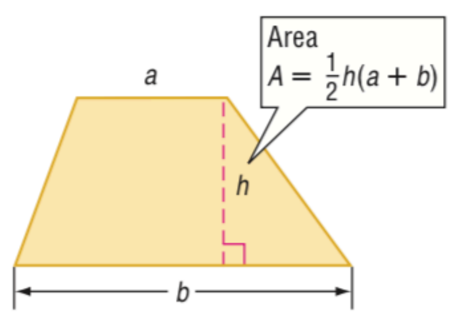
The acceleration due to gravity on the moon is 1.64 meters per second squared.

GEOMETRY **For Exercises 10–12, use the formula for the area of a triangle.**



10. Find the area of a triangle with a base of 18 feet and a height of 7 feet.
11. Solve the formula for h.
12. What is the height of a triangle with area of 28 square feet and base of 8 feet?

GEOMETRY **For Exercises 34 and 35, use the formula for the area of a trapezoid.**



34. Solve the formula for h.
35. What is the height of a trapezoid with an area of 60 square meters and bases of 8 meters and 12 meters?

WORK **For Exercises 36 and 37, use the following information.**

The formula $s = \frac{w - 10e}{m}$ is often used by placement services to find keyboarding speeds. In the formula, s represents the speed in words per minute, w represents the number of words typed, e represents the number of errors, and m represents the number of minutes typed.

36. Solve the formula for e.

37. If Miguel typed 410 words in 5 minutes and received a keyboard speed of 76 words per minute, how many errors did he make?

FLOORING **For Exercises 38 and 39, use the following information.**

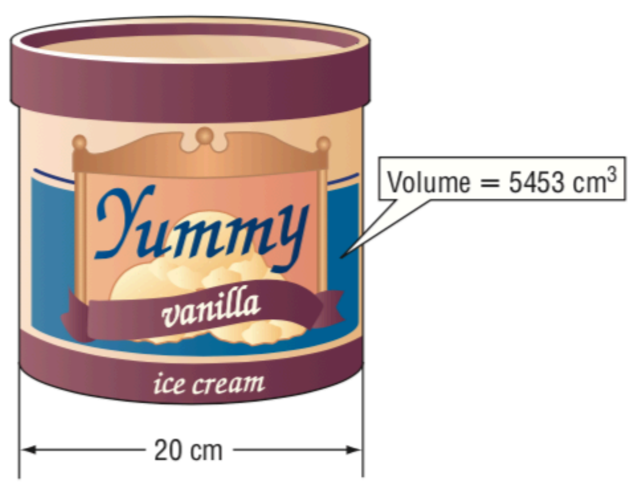
The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by the heel of a shoe. In this formula, P represents the pressure in pounds per square inch, W represents the weight of a person wearing the shoe in pounds, and H is the width of the heel of the shoe in inches.

38. Solve the formula for W.

39. Find the weight of the person if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

40. **ROCKETRY** In the book *October Sky,* high school students were experimenting with different rocket designs. One formula they used was $R = \frac{S + F + P}{S + P}$, which relates the mass ratio R of a rocket to the mass of the structure S, the mass of the fuel F, and the mass of the payload P. The students needed to determine how much fuel to load in the rocket. How much fuel should be loaded in a rocket whose basic structure and payload each have a mass of 900 grams, if the mass ratio is to be 6?

41. **PACKAGING** The Yummy Ice Cream Company wants to package ice cream in cylindrical containers that have a volume of 5453 cubic centimeters. The marketing department decides the diameter of the base of the containers should be 20 centimeters. How tall should the containers be? (*Hint*: $V = \pi r^2 h$)



SOLVING INEQUALITIES

Example 3 Write and Solve an Inequality

Write an inequality for the sentence below. Then solve the inequality.
Three times a number minus eighteen is at least five times the number plus twenty-one.

$$\underbrace{\text{Three times a number}}_{3n}\quad \underbrace{\text{minus}}_{-}\quad \underbrace{\text{eighteen}}_{18}\quad \underbrace{\text{is at least}}_{\geq}\quad \underbrace{\text{five times the number}}_{5n}\quad \underbrace{\text{plus}}_{+}\quad \underbrace{\text{twenty one.}}_{21}$$

$3n - 18 \geq 5n + 21$	Original inequality
$3n - 18 - 5n \geq 5n + 21 - 5n$	Subtract $5n$ from each side.
$-2n - 18 \geq 21$	Simplify.
$-2n - 18 + 18 \geq 21 + 18$	Add 18 to each side.
$-2n \geq 39$	Simplify.
$\frac{-2n}{-2} \leq \frac{39}{-2}$	Divide each side by -2 and change $\geq$ to $\leq$.
$n \leq -19.5$	Simplify.

The solution set is $\{n \mid n \leq -19.5\}$.

3. Justify each indicated step.

$$3(a - 7) + 9 \leq 21$$

$$3a - 21 + 9 \leq 21 \qquad \textbf{a.}\ \underline{\ ?\ }$$

$$3a - 12 \leq 21$$

$$3a - 12 + 12 \leq 21 + 12 \qquad \textbf{b.}\ \underline{\ ?\ }$$

$$3a \leq 33$$

$$\frac{3a}{3} \leq \frac{33}{3} \qquad \textbf{c.}\ \underline{\ ?\ }$$

$$a \leq 11$$

Justify each indicated step.

11. $\frac{2}{5}w + 7 \le -9$

$\frac{2}{5}w + 7 - 7 \le -9 - 7$ **a.** ___?___

$\frac{2}{5}w \le -16$

$\left(\frac{5}{2}\right)\frac{2}{5}w \le \left(\frac{5}{2}\right)(-16)$ **b.** ___?___

$w \le -40$

12. $m > \frac{15 - 2m}{-3}$

$(-3)m < (-3)\frac{15 - 2m}{-3}$ **a.** ___?___

$-3m < 15 - 2m$

$-3m + 2m < 15 - 2m + 2m$ **b.** ___?___

$-m < 15$

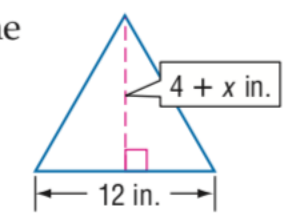
$(-1)(-m) > (-1)15$ **c.** ___?___

$m > -15$

45. **GEOMETRY** The area of a rectangle is less than 85 square feet. The length of the rectangle is 20 feet. What is the width of the rectangle?

49. **LANDSCAPING** Matthew is planning a circular flower garden with a low fence around the border. If he can use up to 38 feet of fence, what radius can he use for the garden? *(Hint*: $C = 2\pi r$)

50. **GEOMETRY** The length of the base of the triangle at the right is less than the height of the triangle. What are the possible values of x?



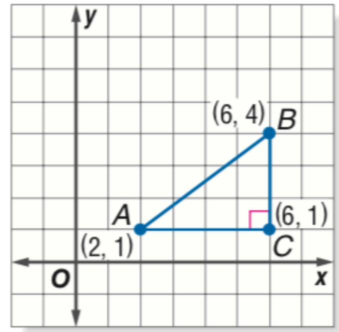
GEOMETRY **For Exercises 39 and 40, use the following information.**
By definition, the measure of any acute angle is less than 90 degrees. Suppose the measure of an acute angle is $3a - 15$.

39. Write an inequality to represent the situation.

40. Solve the inequality.

Example 5 Write an Equation in Point-Slope Form

GEOMETRY **The figure shows right triangle ABC.**



a. Write the point-slope form of the line containing the hypotenuse $\overline{AB}$.

Step 1 First, find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{4 - 1}{6 - 2} \text{ or } \frac{3}{4} \qquad (x_1, y_1) = (2, 1), (x_2, y_2) = (6, 4)$$

Step 2 You can use either point for (x_1, y_1) in the point-slope form.

Method 1 Use (6, 4).

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{3}{4}(x - 6)$$

Method 2 Use (2, 1).

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{4}(x - 2)$$

b. Write each equation in standard form.

$y - 4 = \frac{3}{4}(x - 6)$	Original equation	$y - 1 = \frac{3}{4}(x - 2)$
$4(y - 4) = 4\left(\frac{3}{4}\right)(x - 6)$	Multiply each side by 4.	$4(y - 1) = 4\left(\frac{3}{4}\right)(x - 2)$
$4y - 16 = 3(x - 6)$	Multiply.	$4y - 4 = 3(x - 2)$
$4y - 16 = 3x - 18$	Distributive Property	$4y - 4 = 3x - 6$
$4y = 3x - 2$	Add to each side.	$4y = 3x - 2$
$-3x + 4y = -2$	Subtract $3x$ from each side.	$-3x + 4y = -2$
$3x - 4y = 2$	Multiply each side by -1.	$3x - 4y = 2$

Regardless of which point was used to find the point-slope form, the standard form results in the same equation.

Example 1 Parallel Line Through a Given Point

Write the slope-intercept form of an equation for the line that passes through $(-1, -2)$ and is parallel to the graph of $y = -3x - 2$.

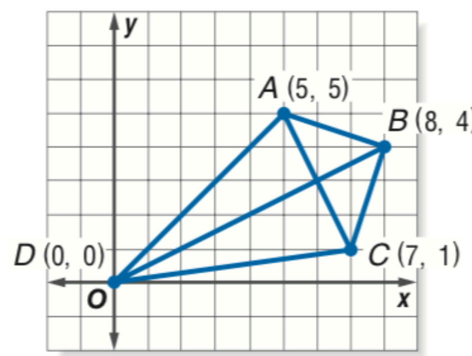
The line parallel to $y = -3x - 2$ has the same slope, -3. Replace m with -3, and (x_1, y_1) with $(-1, -2)$ in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = -3[x - (-1)]$	Replace m with -3, y with -2, and x with -1.
$y + 2 = -3(x + 1)$	Simplify.
$y + 2 = -3x - 3$	Distributive Property
$y + 2 - 2 = -3x - 3 - 2$	Subtract 2 from each side.
$y = -3x - 5$	Write the equation in slope-intercept form.

Therefore, the equation is $y = -3x - 5$.

Example 2 Determine Whether Lines are Perpendicular

KITES **The outline of a kite is shown on a coordinate plane. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.**



Find the slope of each segment.

Slope of $\overline{AC}$: $m = \frac{5-1}{5-7}$ or -2

Slope of $\overline{BD}$: $m = \frac{4-0}{8-0}$ or $\frac{1}{2}$

The line segments are perpendicular because $\frac{1}{2}(-2) = -1$.

Example 3 Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of a line that passes through $(-3, -2)$ and is perpendicular to the graph of $x + 4y = 12$.

Step 1 Find the slope of the given line.

$$\begin{aligned}
x + 4y &= 12 && \text{Original equation} \\
x + 4y - x &= 12 - x && \text{Subtract } 1x \text{ from each side.} \\
4y &= -1x + 12 && \text{Simplify.} \\
\frac{4y}{4} &= \frac{-1x + 12}{4} && \text{Divide each side by 4.} \\
y &= -\frac{1}{4}x + 3 && \text{Simplify.}
\end{aligned}$$

Step 2 The slope of the given line is $-\frac{1}{4}$. So, the slope of the line perpendicular to this line is the opposite reciprocal of $-\frac{1}{4}$, or 4.

Step 3 Use the point-slope form to find the equation.

$$\begin{aligned}
y - y_1 &= m(x - x_1) && \text{Point-slope form} \\
y - (-2) &= 4[x - (-3)] && (x_1, y_1) = (-3, -2) \text{ and } m = 4 \\
y + 2 &= 4(x + 3) && \text{Simplify.} \\
y + 2 &= 4x + 12 && \text{Distributive Property} \\
y + 2 - 2 &= 4x + 12 - 2 && \text{Subtract 2 from each side.} \\
y &= 4x + 10 && \text{Simplify.}
\end{aligned}$$

Therefore, the equation of the line is $y = 4x + 10$.

CHECK You can check your result by graphing both equations on a graphing calculator. Use the CALC menu to verify that $y = 4x + 10$ passes through $(-3, -2)$.

$[-15.16..., 15.16...]$ scl: 1 by $[-10, 10]$ scl: 1

Example 4 Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of a line perpendicular to the graph of $y = -\frac{1}{3}x + 2$ and passes through the x-intercept of that line.

Step 1 Find the slope of the perpendicular line. The slope of the given line is $-\frac{1}{3}$, therefore a perpendicular line has slope 3 because $-\frac{1}{3} \cdot 3 = -1$.

Step 2 Find the x-intercept of the given line.

$y = -\frac{1}{3}x + 2$ Original equation

$0 = -\frac{1}{3}x + 2$ Replace y with 0.

$-2 = -\frac{1}{3}x$ Subtract 2 from each side.

$6 = x$ Multiply each side by -3.

The x-intercept is at (6, 0).

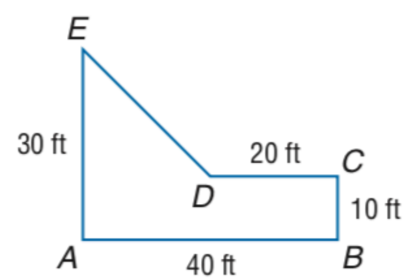
Step 3 Substitute the slope and the given point into the point-slope form of a linear equation. Then write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ Point-slope form

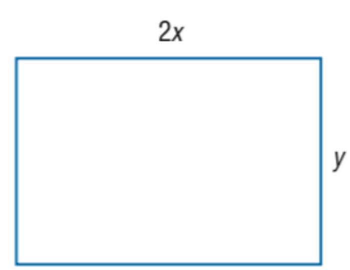
$y - 0 = 3(x - 6)$ Replace x with 6, y with 0, and m with 3.

$y = 3x - 18$ Distributive Property

12. **ARCHITECTURE** Chun Wei has sketched the southern view of a building. If A is located on a coordinate system at $(-40, 10)$, locate the coordinates of the other vertices.



GEOMETRY **For Exercises 46–48, refer to the figure.** The perimeter P of a rectangle is given by $2\ell + 2w = P$, where ℓ is the length of the rectangle and w is the width.



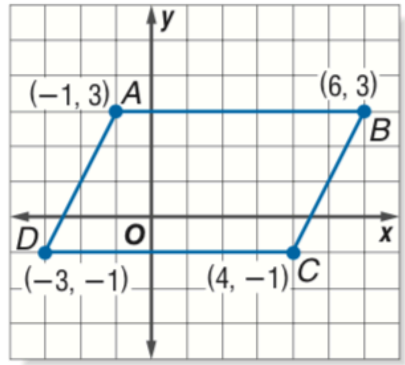
46. If the perimeter of the rectangle is 30 inches, write an equation for the perimeter in standard form.

47. What are the x- and y-intercepts of the graph of the equation?

48. Graph the equation.

GEOMETRY For Exercises 13 and 14, use parallelogram $ABCD$.

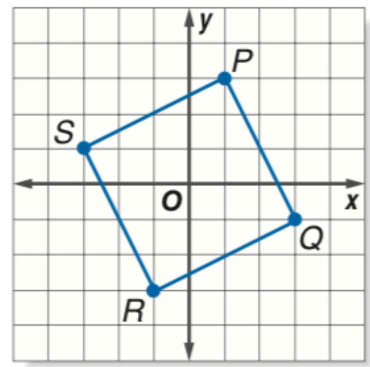
A parallelogram has opposite sides parallel.



13. Write the point-slope form of the line containing $\overline{AD}$.

14. Write the standard form of the line containing $\overline{AD}$.

GEOMETRY For Exercises 61–63, use square $PQRS$.



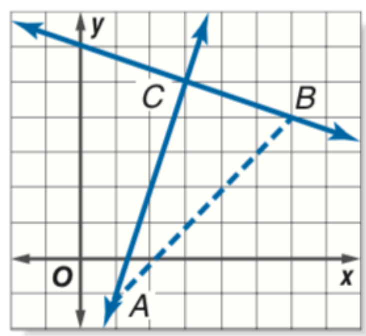
61. Write a point-slope equation of the line containing each side.

62. Write the slope-intercept form of each equation.

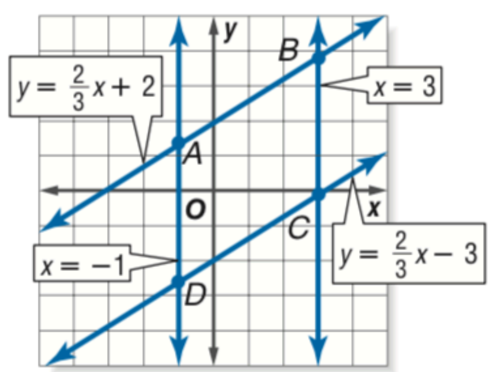
63. Write the standard form of each equation.

8. **GEOMETRY** Quadrilateral $ABCD$ has vertices $A(-2, 1)$, $B(3, 3)$, $C(5, 7)$, and $D(0, 5)$. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.

12. **GEOMETRY** The line with equation $y = 3x - 4$ contains side $\overline{AC}$ of right triangle ABC. If the vertex of the right angle C is at $(3, 5)$, what is an equation of the line that contains side $\overline{BC}$?



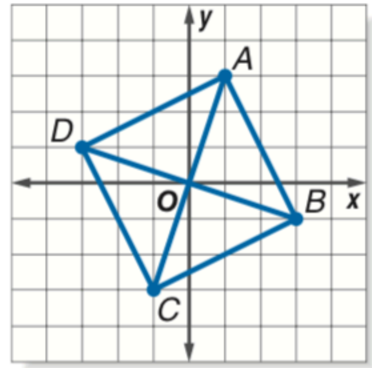
25. **GEOMETRY** A *parallelogram* is a quadrilateral in which opposite sides are parallel. Is $ABCD$ a parallelogram? Explain.



26. Write an equation of the line parallel to the graph of $y = 5x - 3$ and through the origin.

27. Write an equation of the line that has y-intercept -6 and is parallel to the graph of $x - 3y = 8$.

45. **GEOMETRY** The diagonals of a square are segments that connect the opposite vertices. Determine the relationship between the diagonals $\overline{AC}$ and $\overline{BD}$ of square $ABCD$.



SYSTEM OF EQUATIONS

Example 1 Solve Using Substitution

Use substitution to solve the system of equations.

$\mathbf{y = 3x}$
$\mathbf{x + 2y = -21}$

Since $y = 3x$, substitute $3x$ for y in the second equation.

$x + 2y = -21$	Second equation
$x + 2(3x) = -21$	$y = 3x$
$x + 6x = -21$	Simplify.
$7x = -21$	Combine like terms.
$\frac{7x}{7} = \frac{-21}{7}$	Divide each side by 7.
$x = -3$	Simplify.

Use $y = 3x$ to find the value of y.

$y = 3x$	First equation
$y = 3(-3)$	$x = -3$
$y = -9$	The solution is $(-3, -9)$.

Now, substitute the value for y in either original equation and solve for x.

$x + 2y = 8$	First equation
$x + 2(-7) = 8$	Replace y with -7.
$x - 14 = 8$	Simplify.
$x = 22$	

The solution of the system is $(22, -7)$.

Example 2 Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = -25$$
$$2x - 3y = 6$$

Method 1 Eliminate x.

$3x + 4y = -25$ Multiply by 2. $\rightarrow$ $6x + 8y = -50$

$2x - 3y = 6$ Multiply by -3. $\rightarrow$ $(+)\ -6x + 9y = -18$

$$17y = -68 \quad \text{Add the equations.}$$
$$\frac{17y}{17} = \frac{-68}{17} \quad \text{Divide each side by 17.}$$
$$y = -4 \quad \text{Simplify.}$$

Now substitute -4 for y in either equation to find the value of x.

$2x - 3y = 6$	Second equation
$2x - 3(-4) = 6$	$y = -4$
$2x + 12 = 6$	Simplify.
$2x + 12 - 12 = 6 - 12$	Subtract 12 from each side.
$2x = -6$	Simplify.
$\frac{2x}{2} = \frac{-6}{2}$	Divide each side by 2.
$x = -3$	Simplify.

The solution is $(-3, -4)$.

Method 2 Eliminate y.

$3x + 4y = -25$ Multiply by 3. $\rightarrow$ $9x + 12y = -75$

$2x - 3y = 6$ Multiply by 4. $\rightarrow$ $(+)\ 8x - 12y = 24$

$$17x = -51 \quad \text{Add the equations.}$$
$$\frac{17x}{17} = \frac{-51}{17} \quad \text{Divide each side by 17.}$$
$$x = -3 \quad \text{Simplify.}$$

Now substitute -3 for x in either equation to find the value of y.

$2x - 3y = 6$	Second equation
$2(-3) - 3y = 6$	$x = -3$
$-6 - 3y = 6$	Simplify.
$-6 - 3y + 6 = 6 + 6$	Add 6 to each side.
$-3y = 12$	Simplify.
$\frac{-3y}{-3} = \frac{12}{-3}$	Divide each side by -3.
$y = -4$	Simplify.

The solution is $(-3, -4)$, which matches the result obtained with Method 1.

Example 2 Write and Solve a System of Equations

Twice one number added to another number is 18. Four times the first number minus the other number is 12. Find the numbers.

Let x represent the first number and y represent the second number.

$$\underbrace{\text{Twice one number}}_{2x} \quad \underbrace{\text{added to}}_{+} \quad \underbrace{\text{another number}}_{y} \quad \underbrace{\text{is}}_{=} \quad \underbrace{18.}_{18}$$

$$\underbrace{\text{Four times the first number}}_{4x} \quad \underbrace{\text{minus}}_{-} \quad \underbrace{\text{the other number}}_{y} \quad \underbrace{\text{is}}_{=} \quad \underbrace{12.}_{12}$$

Use elimination to solve the system.

$$\begin{array}{rl} 2x + y = 18 & \\ (+)\ 4x - y = 12 & \\ \hline 6x \quad = 30 & \end{array}$$

Write the equations in column form and add.

Notice that the variable y is eliminated.

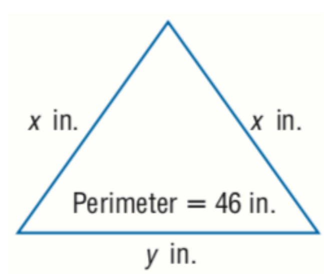
$\frac{6x}{6} = \frac{30}{6}$ Divide each side by 6.

$x = 5$ Simplify.

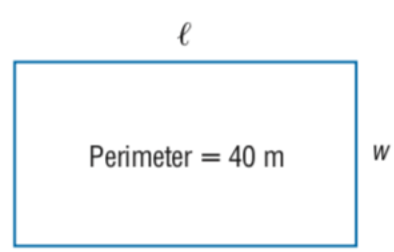
Now substitute 5 for x in either equation to find the value of y.

$4x - y = 12$	Second equation
$4(5) - y = 12$	Replace x with 5.
$20 - y = 12$	Simplify.
$20 - y - 20 = 12 - 20$	Subtract 20 from each side.
$-y = -8$	Simplify.
$\frac{-y}{-1} = \frac{-8}{-1}$	Divide each side by -1.
$y = 8$	The numbers are 5 and 8.

29. GEOMETRY The base of the triangle is 4 inches longer than the length of one of the other sides. Use a system of equations to find the length of each side of the triangle.



41. GEOMETRY The length of the rectangle at the right is 1 meter less than twice its width. What are the dimensions of the rectangle?



GEOMETRY **For Exercises 42 and 43, use the graphs of $y = 2x + 6$, $3x + 2y = 19$, and $y = 2$, which contain the sides of a triangle.**

42. Find the coordinates of the vertices of the triangle.

43. Find the area of the triangle.

32. **GEOMETRY** Supplementary angles are two angles whose measures have the sum of 180 degrees. Angles X and Y are supplementary, and the measure of angle X is 24 degrees greater than the measure of angle Y. Find the measures of angles X and Y.

LAWS OF EXPONENTS

Example 2 Product of Powers

Simplify each expression.

a. $(5x^7)(x^6)$

$(5x^7)(x^6) = (5)(1)(x^7 \cdot x^6)$	Commutative and Associative Properties
$= (5 \cdot 1)(x^{7+6})$	Product of Powers
$= 5x^{13}$	Simplify.

b. $(4ab^6)(-7a^2b^3)$

$(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3)$	Commutative and Associative Properties
$= -28(a^{1+2})(b^{6+3})$	Product of Powers
$= -28a^3b^9$	Simplify.

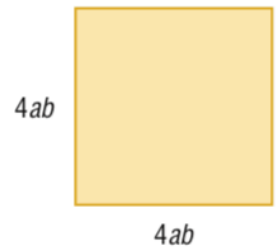
Example 3 Power of a Power

Simplify $((3^2)^3)^2$.

$((3^2)^3)^2 = (3^{2 \cdot 3})^2$	Power of a Power
$= (3^6)^2$	Simplify.
$= 3^{6 \cdot 2}$	Power of a Power
$= 3^{12}$ or 531,441	Simplify.

Example 4 Power of a Product

GEOMETRY **Express the area of the square as a monomial.**



$$\begin{aligned} \text{Area} &= s^2 && \text{Formula for the area of a square} \\ &= (4ab)^2 && s = 4ab \\ &= 4^2a^2b^2 && \text{Power of a Product} \\ &= 16a^2b^2 && \text{Simplify.} \end{aligned}$$

The area of the square is $16a^2b^2$ square units.

Example 1 Quotient of Powers

Simplify $\frac{a^5b^8}{ab^3}$. Assume that a and b are not equal to zero.

$$\begin{aligned} \frac{a^5b^8}{ab^3} &= \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right) && \text{Group powers that have the same base.} \\ &= (a^{5-1})(b^{8-3}) && \text{Quotient of Powers} \\ &= a^4b^5 && \text{Simplify.} \end{aligned}$$

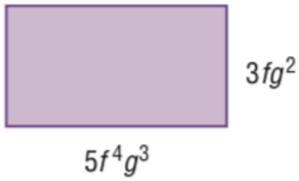
Example 2 Power of a Quotient

Simplify $\left(\frac{2p^2}{3}\right)^4$.

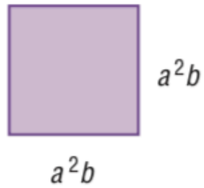
$$\begin{aligned} \left(\frac{2p^2}{3}\right)^4 &= \frac{(2p^2)^4}{3^4} && \text{Power of a Quotient} \\ &= \frac{2^4(p^2)^4}{3^4} && \text{Power of a Product} \\ &= \frac{16p^8}{81} && \text{Power of a Power} \end{aligned}$$

GEOMETRY **Express the area of each figure as a monomial.**

43.



44.



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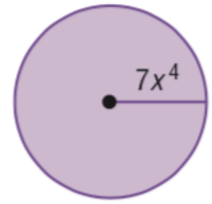
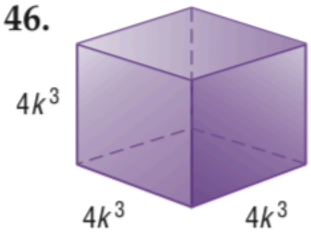
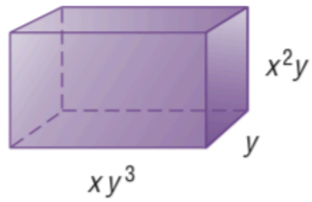


GEOMETRY **Express the volume of each solid as a monomial.**

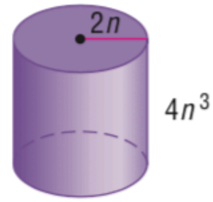
46.



47.

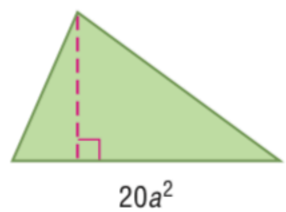


48.



38. The area of the rectangle is $24x^5y^3$ square units. Find the length of the rectangle.

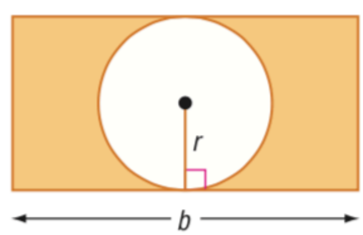
39. The area of the triangle is $100a^3b$ square units. Find the height of the triangle.



POLYNOMIALS

Example 2 Write a Polynomial

GEOMETRY **Write a polynomial to represent the area of the shaded region.**



Words The area of the shaded region is the area of the rectangle minus the area of the circle.

Variables area of shaded region $= A$
width of rectangle $= 2r$
rectangle area $= b(2r)$
circle area $= \pi r^2$

Equation

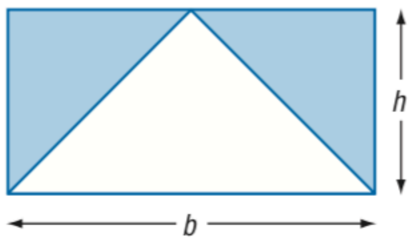
area of shaded region = rectangle area − circle area

$$A = b(2r) - \pi r^2$$
$$A = 2br - \pi r^2$$

The polynomial representing the area of the shaded region is $2br - \pi r^2$.

GEOMETRY **Write a polynomial to represent the area of each shaded region.**

21.



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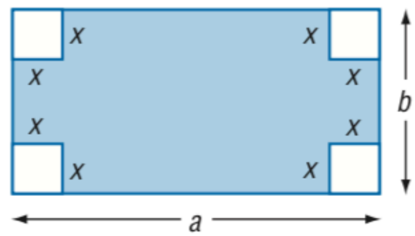

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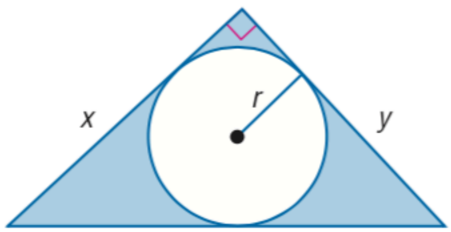

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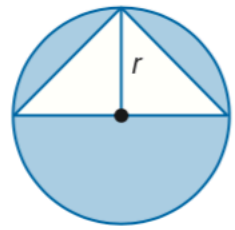
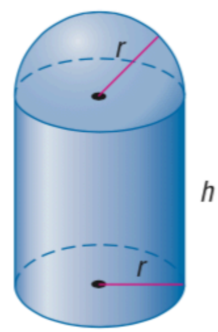


PACKAGING **For Exercises 55 and 56, use the following information.**

A convenience store sells milkshakes in cups with semispherical lids. The volume of a cylinder is the product of π, the square of the radius r, and the height h. The volume of a sphere is the product of $\frac{4}{3}$, π, and the cube of the radius.



55. Write a polynomial that represents the volume of the container.

56. If the height of the container is 6 inches and the radius is 2 inches, find the volume of the container.

OPERATIONS WITH POLYNOMIALS

Example 1 Add Polynomials

Find $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$.

Method 1 Horizontal

Group like terms together.

$(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$

$= [3x^2 + (-7x^2)] + (-4x + 2x) + [8 + (-5)]$ — Associative and Commutative Properties

$= -4x^2 - 2x + 3$ — Add like terms.

Method 2 Vertical

Align the like terms in columns and add.

$$\begin{array}{rrrr} & 3x^2 & - 4x & + 8 \\ (+) & -7x^2 & + 2x & - 5 \\ \hline & -4x^2 & - 2x & + 3 \end{array}$$

Notice that terms are in descending order with like terms aligned.

Example 2 Subtract Polynomials

Find $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$.

Method 1 Horizontal

Subtract $7n + 4n^3$ by adding its additive inverse.

$(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$

$= (3n^2 + 13n^3 + 5n) + (-7n - 4n^3)$ — The additive inverse of $7n + 4n^3$ is $-7n - 4n^3$.

$= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)]$ — Group like terms.

$= 3n^2 + 9n^3 - 2n$ — Add like terms.

Method 2 Vertical

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rrrr} & 3n^2 & + 13n^3 & + 5n \\ (-) & & 4n^3 & + 7n \\ \hline \end{array}$$

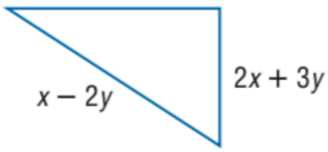
Add the opposite.

$$\begin{array}{rrrr} & 3n^2 & + 13n^3 & + 5n \\ (+) & & -4n^3 & - 7n \\ \hline & 3n^2 & + \; 9n^3 & - 2n \end{array}$$

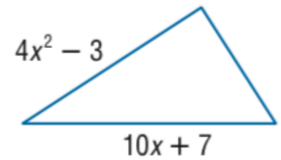
Thus, $(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n$ or, arranged in descending order, $9n^3 + 3n^2 - 2n$.

GEOMETRY **The measures of two sides of a triangle are given. If P is the perimeter, find the measure of the third side.**

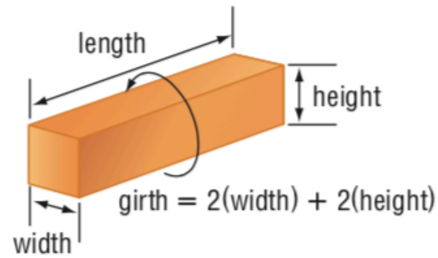
30. $P = 7x + 3y$



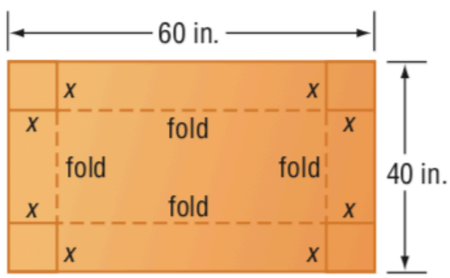
31. $P = 10x^2 - 5x + 16$



POSTAL SERVICE **For Exercises 36–40, use the information below and in the figure at the right.** The U.S. Postal Service restricts the sizes of boxes shipped by parcel post. The sum of the length and the girth of the box must not exceed 108 inches.



Suppose you want to make an open box using a 60-by-40 inch piece of cardboard by cutting squares out of each corner and folding up the flaps. The lid will be made from another piece of cardboard. You do not know how big the squares should be, so for now call the length of the side of each square x.



36. Write a polynomial to represent the length of the box formed.

37. Write a polynomial to represent the width of the box formed.

38. Write a polynomial to represent the girth of the box formed.

39. Write and solve an inequality to find the least possible value of x you could use in designing this box so it meets postal regulations.

40. What is the greatest integral value of x you could use to design this box if it does not have to meet regulations?

MORE OPERATIONS WITH POLYNOMIALS

Example 1 Multiply a Polynomial by a Monomial

Find $-2x^2(3x^2 - 7x + 10)$.

Method 1 Horizontal

$$
\begin{aligned}
&-2x^2(3x^2 - 7x + 10) \\
&= -2x^2(3x^2) - (-2x^2)(7x) + (-2x^2)(10) && \text{Distributive Property} \\
&= -6x^4 - (-14x^3) + (-20x^2) && \text{Multiply.} \\
&= -6x^4 + 14x^3 - 20x^2 && \text{Simplify.}
\end{aligned}
$$

Method 2 Vertical

$$
\begin{array}{lr}
 & 3x^2 - 7x + 10 \\
(\times) & -2x^2 \\
\hline
 & -6x^4 + 14x^3 - 20x^2
\end{array}
\qquad
\begin{array}{l}
\\
\text{Distributive Property} \\
\text{Multiply.}
\end{array}
$$

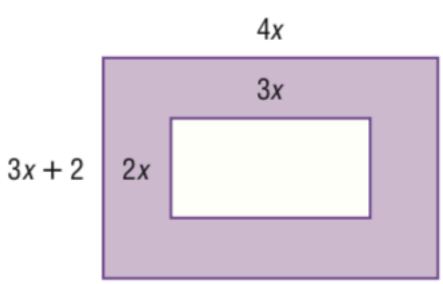
Example 2 Simplify Expressions

Simplify $4(3d^2 + 5d) - d(d^2 - 7d + 12)$.

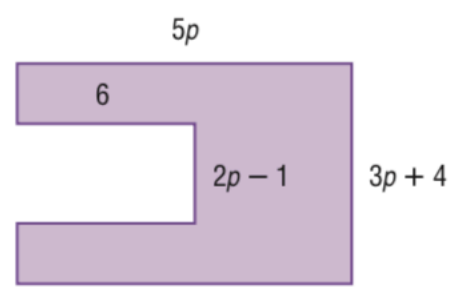
$$
\begin{aligned}
&4(3d^2 + 5d) - d(d^2 - 7d + 12) \\
&= 4(3d^2) + 4(5d) + (-d)(d^2) - (-d)(7d) + (-d)(12) && \text{Distributive Property} \\
&= 12d^2 + 20d + (-d^3) - (-7d^2) + (-12d) && \text{Product of Powers} \\
&= 12d^2 + 20d - d^3 + 7d^2 - 12d && \text{Simplify.} \\
&= -d^3 + (12d^2 + 7d^2) + (20d - 12d) && \text{Commutative and Associative Properties} \\
&= -d^3 + 19d^2 + 8d && \text{Combine like terms.}
\end{aligned}
$$

GEOMETRY **Find the area of each shaded region in simplest form.**

37.

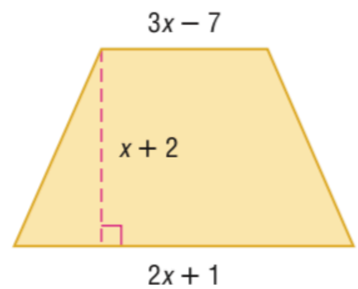


38.



Example 3 FOIL Method

GEOMETRY The area A of a trapezoid is one-half the height h times the sum of the bases, b_1 and b_2. Write an expression for the area of the trapezoid.



Identify the height and bases.

$h = x + 2$

$b_1 = 3x - 7$

$b_2 = 2x + 1$

Now write and apply the formula.

$$\underbrace{A}_{\text{Area}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{\frac{1}{2}}_{\text{one-half}} \cdot \underbrace{h}_{\text{height}} \quad \underbrace{\cdot}_{\text{times}} \quad \underbrace{(b_1 + b_2)}_{\text{sum of bases}}$$

$A = \frac{1}{2}h(b_1 + b_2)$	Original formula
$= \frac{1}{2}(x + 2)[(3x - 7) + (2x + 1)]$	Substitution
$= \frac{1}{2}(x + 2)(5x - 6)$	Add polynomials in the brackets.
$= \frac{1}{2}[x(5x) + x(-6) + 2(5x) + 2(-6)]$	FOIL method
$= \frac{1}{2}(5x^2 - 6x + 10x - 12)$	Multiply.
$= \frac{1}{2}(5x^2 + 4x - 12)$	Combine like terms.
$= \frac{5}{2}x^2 + 2x - 6$	Distributive Property

The area of the trapezoid is $\frac{5}{2}x^2 + 2x - 6$ square units.

Example 4 The Distributive Property

Find each product.

a. $(4x + 9)(2x^2 - 5x + 3)$

$(4x + 9)(2x^2 - 5x + 3)$

$= 4x(2x^2 - 5x + 3) + 9(2x^2 - 5x + 3)$ Distributive Property

$= 8x^3 - 20x^2 + 12x + 18x^2 - 45x + 27$ Distributive Property

$= 8x^3 - 2x^2 - 33x + 27$ Combine like terms.

b. $(y^2 - 2y + 5)(6y^2 - 3y + 1)$

$(y^2 - 2y + 5)(6y^2 - 3y + 1)$

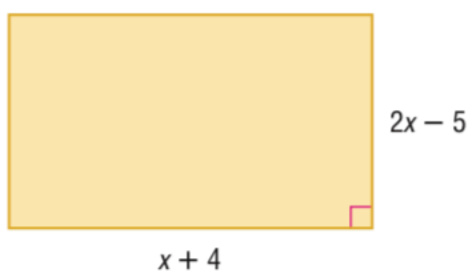
$= y^2(6y^2 - 3y + 1) - 2y(6y^2 - 3y + 1) + 5(6y^2 - 3y + 1)$ Distributive Property

$= 6y^4 - 3y^3 + y^2 - 12y^3 + 6y^2 - 2y + 30y^2 - 15y + 5$ Distributive Property

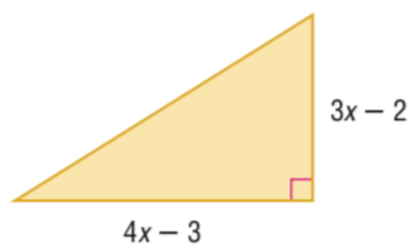
$= 6y^4 - 15y^3 + 37y^2 - 17y + 5$ Combine like terms.

GEOMETRY **Write an expression to represent the area of each figure.**

39.



40.



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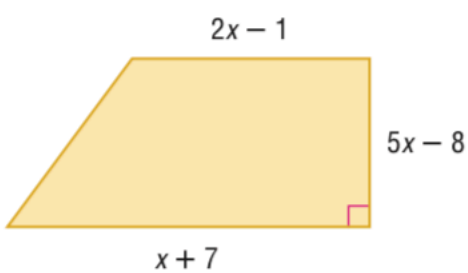

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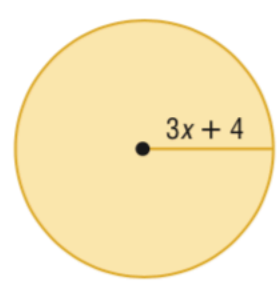
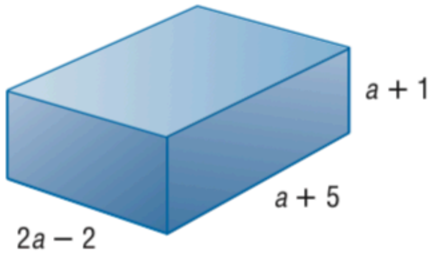
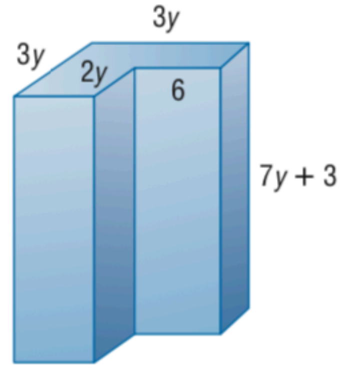


GEOMETRY **The volume V of a prism equals the area of the base B times the height h. Write an expression to represent the volume of each prism.**

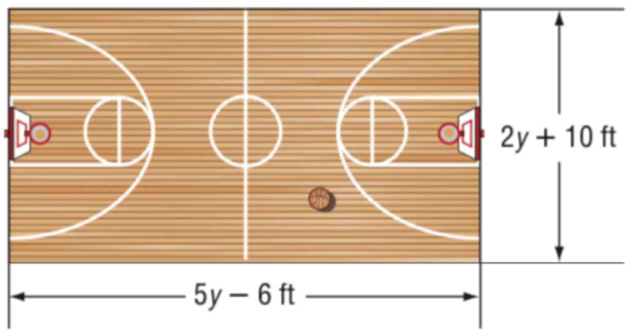
43.



44.



48. **BASKETBALL** The dimensions of a professional basketball court are represented by a width of $2y + 10$ feet and a length of $5y - 6$ feet. Find an expression for the area of the court.



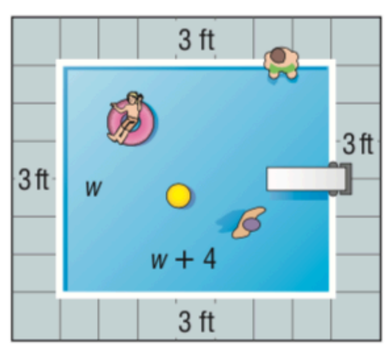
OFFICE SPACE **For Exercises 49–51, use the following information.**
Latanya's modular office is square. Her office in the company's new building will be 2 feet shorter in one direction and 4 feet longer in the other.

49. Write expressions for the dimensions of Latanya's new office.

50. Write a polynomial expression for the area of her new office.

51. Suppose her office is presently 9 feet by 9 feet. Will her new office be bigger or smaller than her old office and by how much?

53. **POOL CONSTRUCTION** A homeowner is installing a swimming pool in his backyard. He wants its length to be 4 feet longer than its width. Then he wants to surround it with a concrete walkway 3 feet wide. If he can only afford 300 square feet of concrete for the walkway, what should the dimensions of the pool be?



FACTORING

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $12a^2 + 16a$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$ Factor each number.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$ Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a$ or $4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$12a^2 + 16a = 4a(3 \cdot a) + 4a(2 \cdot 2)$ Rewrite each term using the GCF.

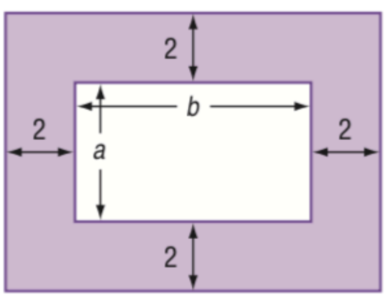
$= 4a(3a) + 4a(4)$ Simplify remaining factors.

$= 4a(3a + 4)$ Distributive Property

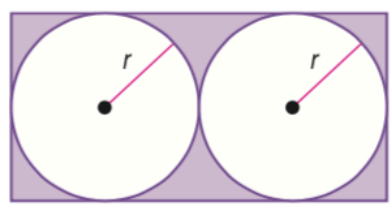
Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

GEOMETRY **Write an expression in factored form for the area of each shaded region.**

44.



45.



GEOMETRY **Find an expression for the area of a square with the given perimeter.**

46. $P = 12x + 20y$ in.

47. $P = 36a - 16b$ cm

RADICALS

Example 1 Simplify Square Roots

Simplify.

a. $\sqrt{12}$

$\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3}$ Prime factorization of 12

$= \sqrt{2^2} \cdot \sqrt{3}$ Product Property of Square Roots

$= 2\sqrt{3}$ Simplify.

b. $\sqrt{90}$

$\sqrt{90} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5}$ Prime factorization of 90

$= \sqrt{3^2} \cdot \sqrt{2 \cdot 5}$ Product Property of Square Roots

$= 3\sqrt{10}$ Simplify.

Example 2 Multiply Square Roots

Find $\sqrt{3} \cdot \sqrt{15}$.

$\sqrt{3} \cdot \sqrt{15} = \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5}$ Product Property of Square Roots

$= \sqrt{3^2} \cdot \sqrt{5}$ Product Property

$= 3\sqrt{5}$ Simplify.

Example 4 Rationalizing the Denominator

Simplify.

a. $\dfrac{\sqrt{10}}{\sqrt{3}}$

$$\frac{\sqrt{10}}{\sqrt{3}} = \frac{\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{\sqrt{30}}{3} \quad \text{Product Property of Square Roots}$$

b. $\dfrac{\sqrt{7x}}{\sqrt{8}}$

$$\frac{\sqrt{7x}}{\sqrt{8}} = \frac{\sqrt{7x}}{\sqrt{2 \cdot 2 \cdot 2}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{7x}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{14x}}{4} \quad \text{Product Property of Square Roots}$$

c. $\dfrac{\sqrt{2}}{\sqrt{6}}$

$$\frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \quad \text{Multiply by } \frac{\sqrt{6}}{\sqrt{6}}.$$

$$= \frac{\sqrt{12}}{6} \quad \text{Product Property of Square Roots}$$

$$= \frac{\sqrt{2 \cdot 2 \cdot 3}}{6} \quad \text{Prime factorization}$$

$$= \frac{2\sqrt{3}}{6} \quad \sqrt{2^2} = 2$$

$$= \frac{\sqrt{3}}{3} \quad \text{Divide the numerator and denominator by 2.}$$

Example 1 Expressions with Like Radicands

Simplify each expression.

a. $4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3}$

$$4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} = (4 + 6 - 5)\sqrt{3} \quad \text{Distributive Property}$$

$$= 5\sqrt{3} \quad \text{Simplify.}$$

b. $12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5}$

$$12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5} = 12\sqrt{5} - 8\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} \quad \text{Commutative Property}$$

$$= (12 - 8)\sqrt{5} + (3 + 6)\sqrt{7} \quad \text{Distributive Property}$$

$$= 4\sqrt{5} + 9\sqrt{7} \quad \text{Simplify.}$$

Example 2 Expressions with Unlike Radicands

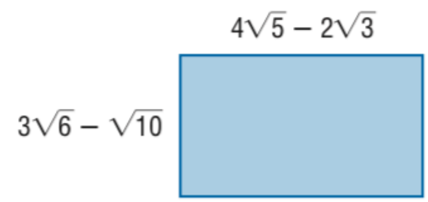
Simplify $2\sqrt{20} + 3\sqrt{45} + \sqrt{180}$.

$$\begin{aligned}
2\sqrt{20} + 3\sqrt{45} + \sqrt{180} &= 2\sqrt{2^2 \cdot 5} + 3\sqrt{3^2 \cdot 5} + \sqrt{6^2 \cdot 5} \\
&= 2\left(\sqrt{2^2} \cdot \sqrt{5}\right) + 3\left(\sqrt{3^2} \cdot \sqrt{5}\right) + \sqrt{6^2} \cdot \sqrt{5} \\
&= 2\left(2\sqrt{5}\right) + 3\left(3\sqrt{5}\right) + 6\sqrt{5} \\
&= 4\sqrt{5} + 9\sqrt{5} + 6\sqrt{5} \\
&= 19\sqrt{5}
\end{aligned}$$

The simplified form is $19\sqrt{5}$.

Example 3 Multiply Radical Expressions

Find the area of the rectangle in simplest form.



To find the area of the rectangle multiply the measures of the length and width.

$$\left(4\sqrt{5} - 2\sqrt{3}\right)\left(3\sqrt{6} - \sqrt{10}\right)$$

$$\begin{aligned}
&= \overbrace{\left(4\sqrt{5}\right)\left(3\sqrt{6}\right)}^{\text{First terms}} + \overbrace{\left(4\sqrt{5}\right)\left(-\sqrt{10}\right)}^{\text{Outer terms}} + \overbrace{\left(-2\sqrt{3}\right)\left(3\sqrt{6}\right)}^{\text{Inner terms}} + \overbrace{\left(-2\sqrt{3}\right)\left(-\sqrt{10}\right)}^{\text{Last terms}} \\
&= 12\sqrt{30} - 4\sqrt{50} - 6\sqrt{18} + 2\sqrt{30} && \text{Multiply.} \\
&= 12\sqrt{30} - 4\sqrt{5^2 \cdot 2} - 6\sqrt{3^2 \cdot 2} + 2\sqrt{30} && \text{Prime factorization} \\
&= 12\sqrt{30} - 20\sqrt{2} - 18\sqrt{2} + 2\sqrt{30} && \text{Simplify.} \\
&= 14\sqrt{30} - 38\sqrt{2} && \text{Combine like terms.}
\end{aligned}$$

The area of the rectangle is $14\sqrt{30} - 38\sqrt{2}$ square units.

Simplify.

1. $\sqrt{100}$
2. $\sqrt{169}$
3. $\sqrt{50}$
4. $\sqrt{700}$
5. $\frac{3}{\sqrt{5}}$
6. $\frac{8}{\sqrt{32}}$
7. $\sqrt{\frac{5}{8}} \cdot \sqrt{\frac{2}{6}}$
8. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{8}{7}}$
9. $\sqrt{7} \cdot 6\sqrt{3}$
10. $5\sqrt{6} \cdot 2\sqrt{3}$
11. $\sqrt{2}(\sqrt{2} + 3\sqrt{5})$
12. $\sqrt{3}(\sqrt{5} + 2)$

13. $3\sqrt{11} + 6\sqrt{11} - 2\sqrt{11}$
14. $4\sqrt{6} + 3\sqrt{2} - 2\sqrt{5}$
15. $9\sqrt{7} - 4\sqrt{2} + 3\sqrt{2} + 5\sqrt{7}$
16. $2\sqrt{27} - 4\sqrt{12}$
17. $8\sqrt{32} + 4\sqrt{50}$
18. $7\sqrt{98} + 5\sqrt{32} - 2\sqrt{75}$
19. $(3 - \sqrt{7})(3 + \sqrt{7})$
20. $(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{2})$
21. $(\sqrt{5})^2$

INDUCTIVE REASONING

Example 1 Extend a Pattern

Study the pattern below.

1 2 3 4 5

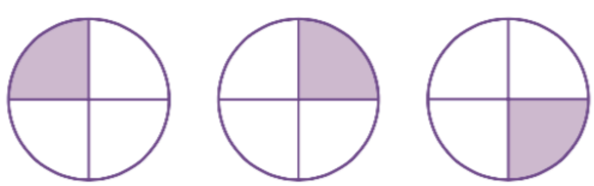
a. Draw the next three figures in the pattern.

The pattern consists of circles with one-fourth shaded. The section that is shaded is rotated in a clockwise direction. The next three figures are shown.

6 7 8

b. Draw the 27th circle in the pattern.

The pattern repeats every fourth design. Therefore designs 4, 8, 12, 16, and so on, will all be the same. Since 24 is the greatest number less than 27 that is a multiple of 4, the 25th circle in the pattern will be the same as the first circle.



25 26 27

Example 2 Patterns in a Sequence

Find the next three terms in the sequence 3, 6, 12, 24, … .

Study the pattern in the sequence.

3 6 12 24

+ 3 + 6 + 12

You can use inductive reasoning to find the next term in a sequence. Notice the pattern 3, 6, 12, … The difference between each term doubles in each successive term. To find the next three terms in the sequence, continue doubling each successive difference. Add 24, 48, and 96.

3 6 12 24 48 96 192

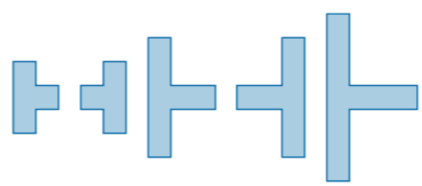
+ 3 + 6 + 12 + 24 + 48 + 96

The next three terms are 48, 96, and 192.

Find the next two items for each pattern.

1.

2.



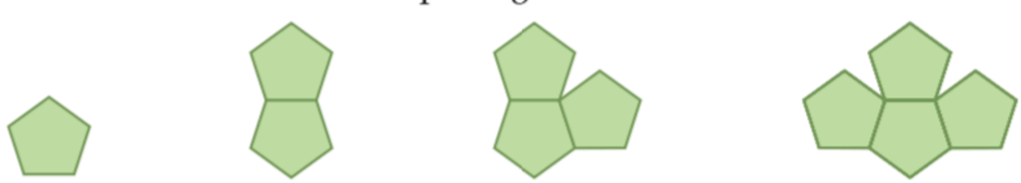
Find the next three terms in each sequence.

3. 12, 23, 34, 45, …

4. 39, 33, 27, 21, …

5. 6.0, 7.2, 8.4, 9.6, …

CRITICAL THINKING **For Exercises 31–33, use the following information.** Suppose you arrange a number of regular pentagons so that only one side of each pentagon touches. Each side of each pentagon is 1 centimeter.



1 pentagon **2 pentagons** **3 pentagons** **4 pentagons**

31. For each arrangement of pentagons, compute the perimeter.

32. Write an equation in function form to represent the perimeter $f(n)$ of n pentagons.

33. What is the perimeter if 24 pentagons are used?

MATRICES

Example 1 Name Dimensions of Matrices

State the dimensions of each matrix. Then identify the position of the circled element in each matrix.

a. $[11 \quad 15 \quad 24]$

This matrix has 1 row and 3 columns. Therefore, it is a 1-by-3 matrix.

The circled element is in the first row and the second column.

b. $\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$

This matrix has 3 rows and 2 columns. Therefore, it is a 3-by-2 matrix.

The circled element is in the third row and the first column.

Example 2 Add Matrices

If $A = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$, find each sum. If the sum does not exist, write *impossible*.

a. $A + B$

$$A + B = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} \qquad \text{Substitution}$$

$$= \begin{bmatrix} 3+7 & -4+(-4) & 7+(-2) \\ -1+1 & 6+6 & 0+(-3) \end{bmatrix} \qquad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 10 & -8 & 5 \\ 0 & 12 & -3 \end{bmatrix} \qquad \text{Simplify.}$$

b. $B + C$

$$B + C = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} \qquad \text{Substitution}$$

Since B is a 2-by-3 matrix and C is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

Example 3 Subtract Matrices

COLLEGE FOOTBALL **The Division I-A college football teams with the five best records during the 1990s are listed below.**

$$\begin{array}{l} \\ \\ \textbf{Florida State} \\ \textbf{Nebraska} \\ \textbf{Marshall} \\ \textbf{Florida} \\ \textbf{Tennessee} \end{array}
\begin{array}{c} \textbf{Overall Record} \\ \begin{array}{ccc} \textbf{Wins} & \textbf{Losses} & \textbf{Ties} \end{array} \\ \begin{bmatrix} 109 & 13 & 1 \\ 108 & 16 & 1 \\ 114 & 25 & 0 \\ 102 & 22 & 1 \\ 99 & 22 & 2 \end{bmatrix} \end{array}
\qquad
\begin{array}{l} \\ \\ \textbf{Florida State} \\ \textbf{Nebraska} \\ \textbf{Marshall} \\ \textbf{Florida} \\ \textbf{Tennessee} \end{array}
\begin{array}{c} \textbf{Bowl Record} \\ \begin{array}{ccc} \textbf{Wins} & \textbf{Losses} & \textbf{Ties} \end{array} \\ \begin{bmatrix} 8 & 2 & 0 \\ 5 & 5 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 0 \\ 6 & 4 & 0 \end{bmatrix} \end{array}$$

Use subtraction of matrices to determine the regular season records of these teams during the decade.

$$\begin{bmatrix} 109 & 13 & 1 \\ 108 & 16 & 1 \\ 114 & 25 & 0 \\ 102 & 22 & 1 \\ 99 & 22 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 2 & 0 \\ 5 & 5 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 0 \\ 6 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 109-8 & 13-2 & 1-0 \\ 108-5 & 16-5 & 1-0 \\ 114-2 & 25-1 & 0-0 \\ 102-5 & 22-4 & 1-0 \\ 99-6 & 22-4 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 101 & 11 & 1 \\ 103 & 11 & 1 \\ 112 & 24 & 0 \\ 97 & 18 & 1 \\ 93 & 18 & 2 \end{bmatrix}$$

So, the regular season records of the teams can be described as follows.

$$\begin{array}{l} \\ \\ \textbf{Florida State} \\ \textbf{Nebraska} \\ \textbf{Marshall} \\ \textbf{Florida} \\ \textbf{Tennessee} \end{array}
\begin{array}{c} \textbf{Regular Season Record} \\ \begin{array}{ccc} \textbf{Wins} & \textbf{Losses} & \textbf{Ties} \end{array} \\ \begin{bmatrix} 101 & 11 & 1 \\ 103 & 11 & 1 \\ 112 & 24 & 0 \\ 97 & 18 & 1 \\ 93 & 18 & 2 \end{bmatrix} \end{array}$$

Example 4 Perform Scalar Multiplication

If $T = \begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix}$**, find** $3T$**.**

$$3T = 3\begin{bmatrix} -4 & 2 \\ 0 & 1 \\ 3 & -6 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 3(-4) & 3(2) \\ 3(0) & 3(1) \\ 3(3) & 3(-6) \end{bmatrix} \quad \text{Definition of scalar multiplication}$$

$$= \begin{bmatrix} -12 & 6 \\ 0 & 3 \\ 9 & -18 \end{bmatrix} \quad \text{Simplify.}$$

State the dimensions of each matrix.

1. $\begin{bmatrix} 1 & 0 & -2 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

If $A = \begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 3 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, **and** $D = \begin{bmatrix} -5 & 1 & -4 \\ -3 & 0 & 2 \end{bmatrix}$,
find each sum, difference, or product. If the sum or difference does not exist, write *impossible*.

5. $A + B$

6. $A + C$

9. $2B$

10. $3C$

13. $2A + C$

14. $3D - B$